

FEASIBILITY OF DETERMINING ATMOSPHERIC OZONE FROM OUTGOING INFRARED ENERGY

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ABSTRACT

The high resolution (5 cm^{-1}) measurements of the outgoing infrared energy in the region of the 9.6μ ozone band offer a means of determining the vertical distribution and total amount of ozone in the earth's atmosphere. With the application of radiative transfer theory and perturbation technique a method is developed to deduce such information. The method hinges on a two-parametric representation of the ozone distribution in the earth's atmosphere.

An error analysis based on four case studies is presented to show how well the atmospheric ozone could be determined. It is found that a small error in radiance value is magnified considerably in the inferred atmospheric ozone.

1. INTRODUCTION

Atmospheric ozone is photochemically produced by solar, near-ultraviolet radiation. The global distribution of this gas as a function of latitude and season can reveal the stratospheric circulation, and coupling between the troposphere and stratosphere. Day-to-day changes in the weather are also reflected in the ozone variations. Thus, it appears that a method of indirectly obtaining the atmospheric ozone from satellite-borne radiation sensors could be very useful.

The problem of indirectly determining the vertical distribution of temperature and water vapor in the earth's atmosphere from remote radiation sensors has been studied by several investigators (see Wark and Fleming, 1966, and also for a review, see Conrath, 1968). Such investigations were concerned with the infrared spectral regions of the 15μ CO_2 band, and the water vapor 6.3μ vibration-rotation and 20μ rotation bands. However, in addition to these bands the IRIS (Infrared Interferometer Spectrometer) spectrum (5 cm^{-1} resolution), obtained at Palestine, Tex., shown in figure 1 reveals marked spectral structure in the region of the 9.6μ ozone band. This feature present in the observed spectrum prompted the present investigation to indirectly determine the atmospheric ozone.

The narrow (about 1.5μ wide) 9.6μ ozone band in such emission spectrum of the earth and atmosphere is influenced by the water vapor absorption in the so-called "window" region extending from 8 to 12μ . Hence, it is necessary to know the distribution of water vapor in the atmosphere in order to interpret the spectrum in the 9.6μ region. Although in principle it is possible to determine the distribution of water vapor, temperature, and ozone indirectly from the radiation measurements, we have not

attempted to do so in this preliminary study. Instead, the distribution of ozone alone is determined from the spectrum when the water vapor and temperature distributions are known.

Earlier studies that attempted to determine the atmospheric ozone from ground-based measurements of the emission spectrum of ozone in the 9.6μ region were not encouraging (see e.g., Dave et al., 1963). However, some disadvantages encountered in interpreting the atmospheric emission spectra when measured at the ground may not be encountered when the spectrum is obtained from space.

With this view in mind, we have computed spectra for several stations at which soundings of temperature, water vapor, and ozone were available. These synthetic spectra were then subjected to an inversion procedure to deduce the ozone distribution. The inversion procedure adopted here depends on a two-parameter model for describing the ozone distribution. Such a model representation is arrived at from broad-based climatological features of the atmospheric ozone.

The feasibility of determining atmospheric ozone is then demonstrated from an error analysis of four case studies. A small finite error in radiance is magnified to a large error in the inferred atmospheric ozone content. The magnification of error is different for each case and is intolerably large under certain circumstances.

Because of several inadequacies of data pertaining to the absorption coefficients and atmospheric temperature and water vapor sounding, we could not perform an inversion and obtain the ozone distribution for Palestine, Tex. However, a reasonable match of the spectrum is shown to illustrate the general soundness of the scheme.

This investigation is intended to be a pilot study of this problem. Several spectra will be available from the forthcoming meteorological satellites, which may be analyzed with the help of the findings from this study.

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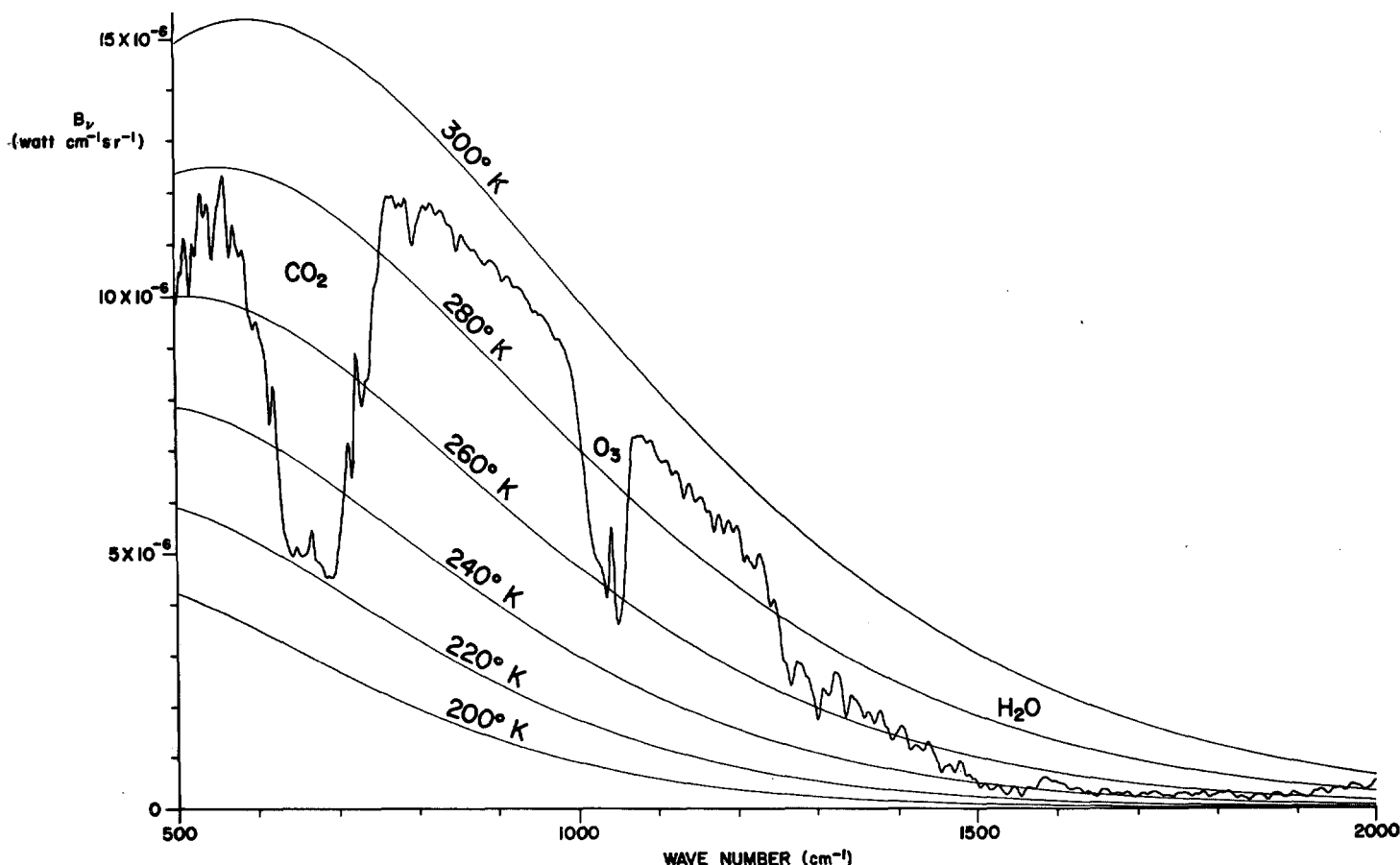


FIGURE 1.—Atmospheric emission spectrum (Bread Board Instrument of IRIS Experiment, GSFC—University of Michigan, balloon altitude 31.4 km) measured at Palestine, Tex., on May 8, 1966.

2. MODEL OF THE ATMOSPHERIC OZONE DISTRIBUTION

The atmospheric ozone is observed to change appreciably from day to day below about 25 km. At higher elevations, where photochemical equilibrium is restored in a day or less, ozone changes are systematic and follow a seasonal variation (Dütsch, 1966). In figure 2, the mean meridional distribution of ozone for spring and fall are shown (after Hering and Borden, 1965b). Some significant features concerning the vertical ozone distribution can be deduced from these meridional cross sections. The pronounced ozone maximum in the stratosphere is present at all latitudes. The position of ozone maximum, which is at about 26 km at low latitudes, drops to lower altitudes as the North Pole is approached. Also, the rather sharp maximum present at low latitudes is replaced by a considerably broad one at high latitudes. The position of the tropopause as a function of latitude has a strong influence on the meridional distribution of ozone. The strong stratification that is present in the stratosphere is replaced by a very weak structure in the troposphere. On a further examination of several ozone soundings, one can observe that the tropopause in the tropical latitudes in all seasons marks the level of transition from strong stratospheric to

weak tropospheric structure in ozone. On the other hand, at middle and high latitudes the tropopause has such significance only in summer and fall. During winter and spring, the transition takes place at a level 3 to 4 km below the tropopause height.

With these climatological features, a model of the distribution of ozone is made involving a minimum number of parameters to describe it. Photochemical equilibrium distribution is assumed above some high altitude z_1 (~ 30 km). Farther down from the level z_1 , to the tropopause height, z_2 , a Gaussian distribution, which can simulate the pronounced maximum, is adopted. Then in the troposphere the ozone concentration is assumed constant, having a value equal to that obtained at z_2 , the tropopause height.

Let the Gaussian distribution describing the ozone variation between z_1 and z_2 be given by

$$O_3(z) = O_3(z_m) \exp\left[-\frac{(z-z_m)^2}{h}\right] \quad (1)$$

where $O_3(z)$ represents the ozone concentration at any height z , ($z_1 > z > z_2$), and $O_3(z_m)$ is the ozone concentration maximum at the height z_m . h is a measure of the dispersion of ozone about the height z_m .

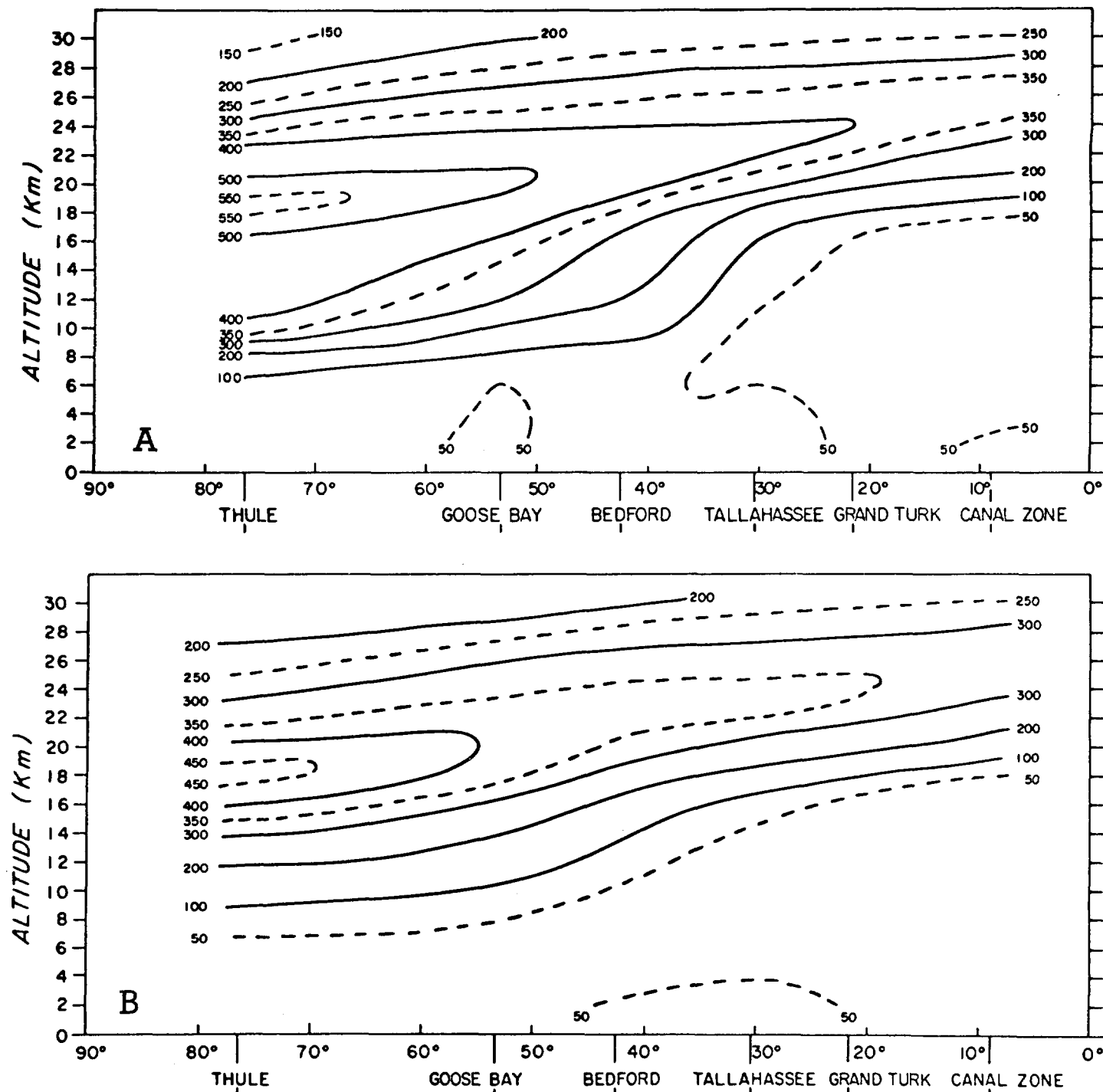


FIGURE 2.—(A) mean ozone density ($\mu\text{gm}/\text{m}^3$) for March, April, and May 1963-1964 (after Hering and Borden, 1965b); (B) mean ozone density ($\mu\text{gm}/\text{m}^3$) for September, October, and November 1963-1964 (after Hering and Borden, 1965b).

Now applying equation (1) at the selected high level z_1 , where $O_3(z_1)$ may be assigned a value, we have

$$O_3(z_1) = O_3(z_m) \exp\left[-\frac{z_1 - z_m}{h}\right]^2. \quad (2)$$

The value of $O_3(z_1)$ may be taken from figure 3 of London (1966).

From equations (1) and (2), $O_3(z_m)$ may be eliminated to yield

$$O_3(z) = O_3(z_1) \exp\left[\frac{z_1 - z_m}{h}\right]^2 \exp\left[-\frac{z - z_m}{h}\right]^2. \quad (3)$$

Equation (3) describes ozone distribution, in the height region z_1 to z_2 , in terms of two unknowns z_m and h . This is essentially the two-parameter representation.

3. TRANSMISSION OF WATER VAPOR AND OZONE IN THE 9.6-MICRON-BAND REGION

In order to compute spectra, as mentioned earlier, or to perform an "inversion" based on the radiative transfer theory, we need to know the transmission of the optically active gases in the atmosphere. The transmission τ_ν in the 9.6- μ -band region is influenced by both water vapor and ozone. Hence the effective transmission is given by

$$\tau_\nu = (\tau_{\text{H}_2\text{O}} \cdot \tau_{\text{O}_3})_\nu \quad (4)$$

where $\tau_{\text{H}_2\text{O}}$ and τ_{O_3} are the transmissions of water vapor and ozone.

Based upon several earlier studies, Möller and Raschke (1963) have presented a method of obtaining the transmission function for the water vapor in the 8–12 μ region. This method is adopted in the present study.

The transmission function for ozone is taken from a study of Walshaw (1954), where he has investigated the 9.6 μ band with a resolution of 6.5 cm^{-1} . Walshaw has fitted the transmission by means of a statistical-band model with allowance for a "modulated contour" for the band (see Goody, 1964a). The transmission function is given by

$$(\tau_{\text{O}_3})_\nu = [\exp(-X_\nu)] \times \left. \begin{array}{l} \frac{\sinh k_\nu^{1/2} X_\nu}{k_\nu^{1/2} X_\nu} \text{ when } k_\nu > 0 \\ \frac{\sin |k_\nu|^{1/2} X_\nu}{|k_\nu|^{1/2} X_\nu} \text{ when } k_\nu < 0 \end{array} \right\} \quad (5)$$

where

$$X_\nu = \left[\frac{\sigma_\nu m}{\delta} \left(1 + \frac{\sigma_\nu m}{\pi \alpha} \right)^{-1/2} \right],$$

σ_ν is the mean-line strength,² α is the mean-line half width, δ is the mean-line spacing, m is the path length of ozone, and k_ν is a constant empirically determined as a function of ν to get the best fit to the experimental measurements. The above relationship is based on the assumption of exponential intensity distribution of spectral lines.

Walshaw's measurements were made for homogeneous paths in the laboratory. In the atmosphere we do not obtain such homogeneity conditions, as the absorbing gas is distributed along a strongly varying pressure path. The variation of temperature along the path, though present in the earth's atmosphere, is never as pronounced. To take into account such inhomogeneous conditions, the Curtis-Godson approximation is commonly used. This approximation is shown to be quite inadequate for the atmospheric ozone by Walshaw and Rodgers (1963) amongst others. It is possible to incorporate such inhomogeneous conditions and calculate the transmission, without any approximation, at each frequency in a single spectral line (see e.g.,

Hitschfeld and Houghton, 1961). With the help of such a procedure we can compute numerically the equivalent width, W_ν , of a spectral line as follows:

$$W_\nu = \int_{-\infty}^{+\infty} \left(1 - \exp \left\{ - \int_{z'}^{z''} \frac{\sigma_\nu}{\pi \alpha^2(z) + (\nu - \nu_0)^2} \text{O}_3(z) dz \right\} \right) d\nu. \quad (6)$$

The equivalent width W_ν is thus a function of the optical path between any height z' and the top of the atmosphere. In the above equation α is the Lorentz half width, ν_0 is the frequency at the center of the line, and O_3 is the ozone concentration.

The dependence of α on the pressure p and the temperature T can be expressed as

$$\alpha(z) = \alpha_0 \left(\frac{p(z)}{p_0} \right) \left(\frac{T_0}{T(z)} \right)^{1/2} \quad (7)$$

where α_0 is the Lorentz half width at standard pressure and temperature p_0 and T_0 , respectively.

If synthetic spectra are to be calculated, O_3 as a function of height may be specified from available vertical ozone soundings. On the other hand, when we wish to solve for $\text{O}_3(z)$ by an inversion method, a guess for the distribution of ozone is provided by equation (3) in terms of z_m and h , together with the model of the atmospheric ozone adopted.

In principle σ_ν , the line strength, is a function of temperature and therefore of height. But we have neglected this temperature dependence, as it is not known.

The equivalent width, W_ν , of a single spectral line so obtained can be used to get the transmission function over a spectral interval wide enough to contain a large number of spectral lines from the relationship (Goody, 1964b) given below:

$$\tau' = \exp \left(- \frac{W_\nu}{\delta} \right). \quad (8)$$

The above expression is based on a statistical model for the transmission function that assumes lines of equal intensity but having random position. We distinguish this transmission with a prime as it is not equal to the transmission one would have if the spectral lines had an exponential intensity distribution. Plass (1958) has shown that it is possible to match the transmission given by the statistical-band model of equal intensity lines with that having lines of exponential intensity distribution. From such a method of matching the two types of statistical-band models, we arrive at the following transmission function for ozone, in the 9.6- μ -band region, which incorporates all inhomogeneities along the absorption path:

$$(\tau_{\text{O}_3})_\nu = [\exp(-X'_\nu)] \times \left. \begin{array}{l} \frac{\sinh k_\nu^{1/2} X'_\nu}{k_\nu^{1/2} X'_\nu} \text{ when } k_\nu > 0 \\ \frac{\sin |k_\nu|^{1/2} X'_\nu}{|k_\nu|^{1/2} X'_\nu} \text{ when } k_\nu < 0 \end{array} \right\} \quad (9)$$

² All the parameters σ_ν , α , δ , and k_ν are obtained from Walshaw (1954). However according to Walshaw (1957), σ_ν was multiplied by 1.30.

where

$$X'_\nu = \frac{\pi W_i}{4 \delta},$$

and the other symbols have the same meaning as before.

4. METHOD OF DETERMINING OZONE DISTRIBUTION

We know from radiative-transfer theory that the intensity I_ν of radiation at frequency ν , upwelling from the earth's atmosphere, is given by the equation (see e.g., Elsasser and Culbertson, 1960)

$$I_\nu = B_\nu(T_0)\tau_\nu(0) + \int_{\tau_\nu(0)}^1 B_\nu(T_z)d\tau_\nu(z) \quad (10)$$

where B_ν is the Planck intensity, T_0 and T_z are the absolute temperatures at the ground and any height z , and $\tau_\nu(0)$ and $\tau_\nu(z)$ are the transmissions from ground and any level z to top of the atmosphere.

Equation (10), which forms the basis of all indirect methods or "inversions," is applied to small but finite, 5 cm^{-1} wide (to correspond to IRIS resolution), spectral intervals of the emission spectrum in the 9.6μ region. As the 9.6μ band of ozone is about 150 cm^{-1} wide, we may think of having about 30 such useful spectral intervals yielding as many independent pieces of information. However, in practice we find that only two independent pieces of information can be elicited. This point will be elaborated in the discussion of the results.

Let us choose two spectral intervals labeled 1 and 2, in the 9.6μ band, for which the upwelling intensities may be written as

$$I_1 = B_1(T_0)\tau_1(0) + \int_{\tau_1(0)}^1 B_1(T_z)d\tau_1(z) \quad (11)$$

and

$$I_2 = B_2(T_0)\tau_2(0) + \int_{\tau_2(0)}^1 B_2(T_z)d\tau_2(z). \quad (12)$$

Since we are assuming the vertical soundings of temperature and water vapor, we need to solve for the two parameters describing the vertical ozone distribution from equations (11) and (12). The information pertaining to ozone is contained in the transmission functions τ_1 and τ_2 through equations (9), (6), and (4).

Let us choose, as an initial guess, some values for z_m and h and then, with the ozone distribution obtained from these parameters as described earlier, compute the intensities I_1' and I_2' in the two spectral intervals from equations (11) and (12). The intensities thus computed may differ from the "observed" intensities I_1 and I_2 by some finite amounts, say, δI_1 and δI_2 . It is now our aim to adjust the two parameters z_m and h by necessary increments δz_m and δh so that the adjusted ozone distribution can yield intensities to match the observed ones. With the known deviations δI_1 and δI_2 , we can hope to solve for the appropriate increments δz_m and δh from a

set of two equations given below:

$$\delta I_1 = \frac{\partial I_1'}{\partial z_m} \delta z_m + \frac{\partial I_1'}{\partial h} \delta h \quad (13)$$

and

$$\delta I_2 = \frac{\partial I_2'}{\partial z_m} \delta z_m + \frac{\partial I_2'}{\partial h} \delta h. \quad (14)$$

The four coefficients $\delta I_1'/\delta z_m$, $\delta I_1'/\delta h$, $\delta I_2'/\delta z_m$, and $\delta I_2'/\delta h$ that are needed to solve the above equations may be readily computed from equations (11) and (12). For example, using equation (11), $\delta I_1'/\delta z_m$ is deduced by calculating the change in intensity due to a small change, say 0.1 km, in the initial guess for z_m while h is unchanged.

The increments δz_m and δh obtained by solving (13) and (14) help us to correct the initial guess for z_m and h . The corrected z_m and h may now be used as an improved guess, and the whole procedure repeated. Such iterations are done until the deviations δI_1 and δI_2 , between the computed and observed intensities, are reduced to a desired magnitude.

A reasonably good initial guess for z_m as a function of latitude is provided by the mean meridional distribution of ozone shown in figures 2a and 2b. It is found to vary from 18 km at high latitudes to 26 km at low latitudes. Similarly, a good initial guess for h obtained from the same figures is 10 km at high latitudes and 6 km at low latitudes. It is assumed here that the latitude of the station at which the ozone inversion is desired is known. With such helpful initial guess for z_m and h we have found not more than four iterations are necessary to get the calculated intensities accurate to within 1 percent of the observed intensities.

We also find the iteration procedure converges, for middle- and high-latitude cases, when the initial guess for z_m and h is within about 3 km of the correct value. However at low latitudes the guess has to be better, within about 1.5 km, to get a convergence. This is on account of the deep tropospheric layer present in the Tropics.

5. DISCUSSION OF RESULTS

The main objective of this study is to examine the feasibility of determining atmospheric ozone. Since we do not at present have spectra taken at more than one station it has been necessary to calculate emission spectra from several, known simultaneous soundings of ozone, temperature, and dewpoint temperature, and then to invert such synthetic spectra to get back the ozone distributions. We have done this with the help of the atmospheric data presented in "Ozonesonde Observations Over North America," by Hering and Borden (1965a). Three such inversions are shown in figures 3, 4, and 5. The temperature and ozone data were extrapolated to 50-km height.

Two spectral intervals, 5 cm^{-1} wide, centered at 1000 cm^{-1} and 1050 cm^{-1} are used in the inversion scheme.

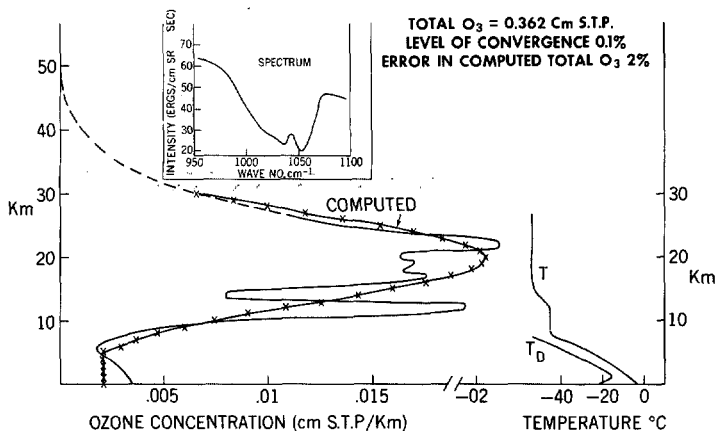


FIGURE 3.—Case study, Bedford, Mass., Feb. 20, 1964.

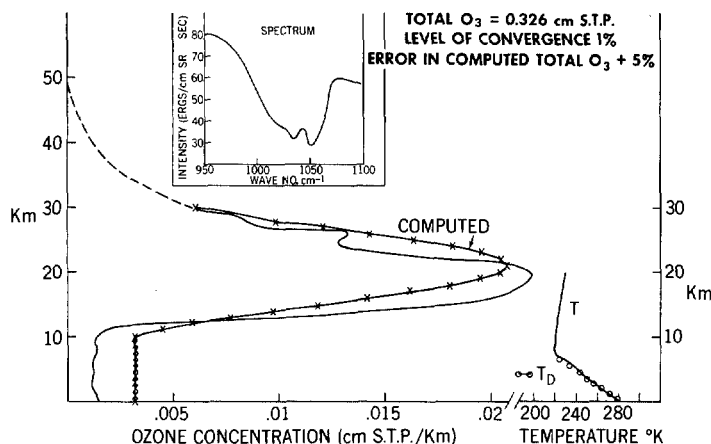


FIGURE 4.—Case study, Fairbanks, Alaska, June 24, 1964.

These spectral intervals are chosen such that one of them at 1000 cm^{-1} has moderate ozone absorption while the other at 1050 cm^{-1} has strong absorption.

A comparison of the computed and observed distribution of ozone at Bedford (fig. 3) shows the ability of the computed curve to approximate smoothly the observed one. Although the computed curve cannot reproduce the double maximum, it succeeds in predicting the total ozone with 2-percent accuracy. This is so because of the extremely small convergence level, 0.1 percent, achieved in matching the radiance computed from observed and calculated ozone curves. In figure 4, we see the solution obtained for an ozone sounding at Fairbanks. Although the observed ozone profile in this case is not as complicated as the one shown for Bedford, the calculated total ozone is off by 5 percent. This is primarily because of the 1-percent convergence level to which the radiances were matched. The computed spectrum for each case is shown in the respective figures. The 9.6μ band in these two cases stands out quite significantly as there is a good amount of stratification in the temperature. To emphasize this aspect we show, in figure 5, a case study for an Arctic station, Thule, Greenland, where the temperature stratification is very weak

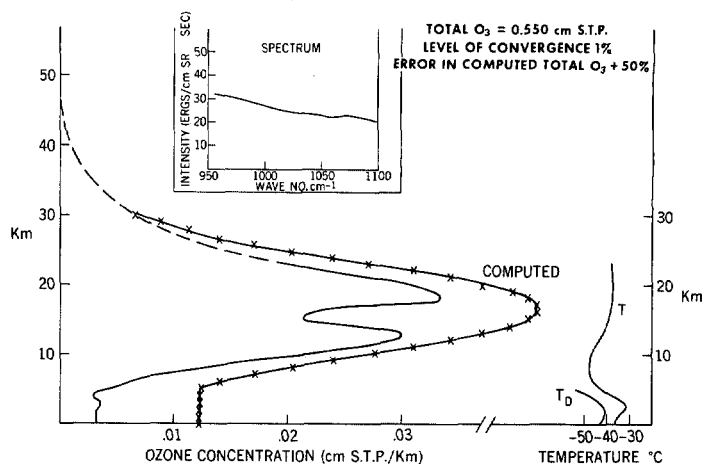


FIGURE 5.—Case study, Thule, Greenland, Mar. 25, 1964.

resulting in almost a black-body continuum for the spectrum. The inverted ozone profile, calculated with such a weak spectrum, shows poor agreement with the observed profile. The computed total ozone is in error by 50 percent.

The observed spectrum at Palestine, Tex., offers a means of testing the inversion scheme. Therefore, an attempt was made to invert the observed radiances in the 9.6μ region of the spectrum and obtain an ozone distribution. This attempt was not successful, for two important reasons. The meteorological data needed for inversion, namely the temperature and dewpoint temperature soundings at Palestine, were not available. Hence, it was necessary to use such data from a neighboring meteorological station, Shreveport. Secondly, the absorption coefficients adopted for water vapor and ozone, although reasonable, are not exact. It is very important to have accurate measurements of the absorption coefficients in order to have a successful inversion scheme. So, as an alternative to a rigorous inversion, we have attempted to match the observed Palestine spectrum with the help of the Shreveport soundings and an ozone profile characteristic of the Tropics shown in figure 6. The observed and computed spectra are compared in figure 7. The lack of agreement between the two spectra may be largely attributed to inaccuracies in the absorption coefficients.

At this point it is desirable to examine the question concerning the number of independent pieces of information about atmospheric ozone that one can possibly elicit out of the emission spectrum in the 9.6μ region. This may be demonstrated from the nature of the weighting functions. In figure 8 we show, for the case of Fairbanks, the weighting functions $d\tau/dz$ for ozone, in the two spectral regions of 5 cm^{-1} width, centered at 1000 and 1050 cm^{-1} . The weighting functions resemble one another and essentially have the shape of the ozone profile. This is so because of the weak absorption in the 9.6μ region. Even in the interval at 1050 cm^{-1} near the center of the absorption band, the ground is "visible" and a significant part of the ground emission is transmitted to the top of the atmosphere. From the strong similarity of the two weighting functions, we may infer that they

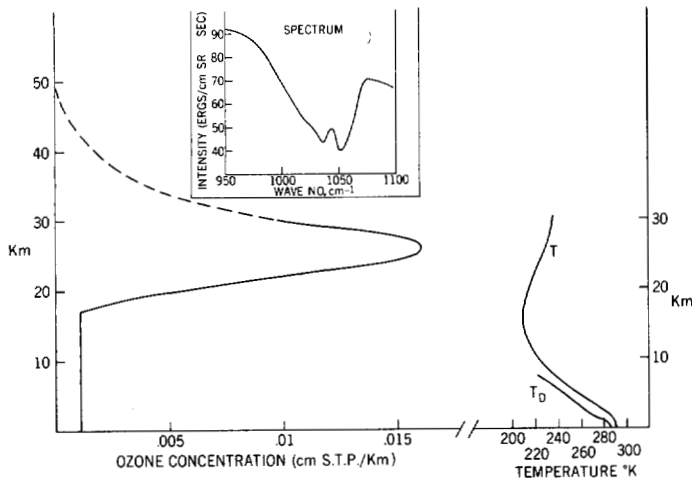


FIGURE 6.—Emission spectrum calculated to match the spectrum obtained at Palestine, Tex., on May 8, 1966.

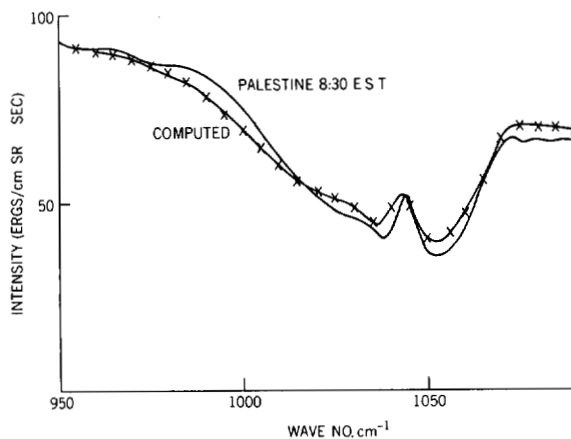


FIGURE 7.—Comparison of the Palestine, Tex., emission spectrum in the 9.6μ region with the calculated spectrum shown in figure 6.

cannot convey substantially different information about the ozone distribution. Nevertheless, a small amount of dissimilarity that is present in the two weighting functions makes the information content of the two spectral intervals to some extent independent of one another. Thus it appears that we can hope to get no more than two independent parameters. In an independent study, Williamson (1968) has come to the same conclusion when the noise level in the spectral measurements is assumed to be about 1 percent. Williamson has used an objective way of finding the information content in the spectrum at 9.6μ region. His method is similar to that of Mateer (1965).

Let us now examine the practicability of determining the atmospheric ozone indirectly from measured radiance in the 9.6μ region. As there are in practice some errors present in the measured radiance, we have to examine the degree to which the deduced solution will be influenced by such errors. For this purpose we have made an analysis of the error introduced in the total ozone due to a known error in radiance.

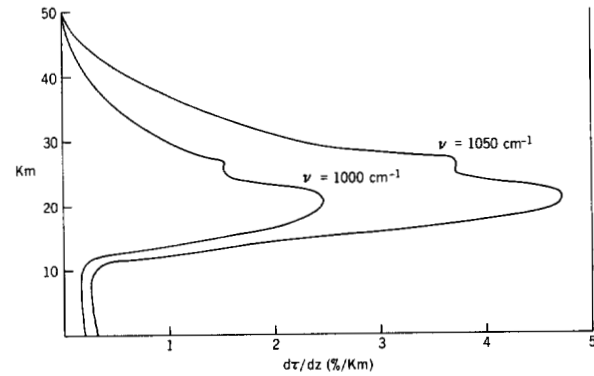


FIGURE 8.—Weighting functions for ozone calculated for the case of Fairbanks, Alaska, June 24, 1964.

TABLE 1.—Amplification of error

Case		$\frac{\partial \log [O_3]}{\partial \log z_m} \cdot \frac{\partial \log z_m}{\partial \log I}$	$\frac{\partial \log [O_3]}{\partial \log h} \cdot \frac{\partial \log h}{\partial \log I}$
Fairbanks	I_1	-3.1	-4.9
	I_2	-2.0	-5.2
Bedford	I_1	-3.7	-4.0
	I_2	-2.6	-3.3
Thule	I_1	-34.5	-105.0
	I_2	-158.0	+104.6
Simulation of Palestine	I_1	-4.3	+2.96
	I_2	-1.5	+0.4

Let the total ozone be denoted by $[O_3]$ and a small error in it by $\delta[O_3]$. Similarly, let I and δI be the corresponding intensity and an error in it. Now since both the total ozone and intensity are functions of the two parameters z_m and h , we seek to get

$$\left(\frac{\Delta \log [O_3]}{\Delta \log z_m} \middle/ \frac{\Delta \log I}{\Delta \log z_m} \right)_h$$

which gives an amplification of the error in total ozone due to an error in intensity when h is held constant. In an analogous way we can get

$$\left(\frac{\Delta \log [O_3]}{\Delta \log h} \middle/ \frac{\Delta \log I}{\Delta \log h} \right)_{z_m}$$

the amplification of error when z_m is a constant. With the help of the four cases presented in figures 3 to 6 we have obtained these amplifications, and they are tabulated for each case and the two wavelengths in table 1. We find from this study that there is generally an amplification of the error. An error of 1 percent in the measured radiance may result in an error of 5 percent in the derived total ozone. In the particular case of Thule, where the atmosphere is close to the isothermal condition, the amplification of error is enormous, and no meaningful information on ozone could be extracted.

The above analysis did not consider the errors introduced by the assumptions made in arriving at a model

for the ozone distribution. The assumption of photochemical equilibrium above some high altitude, 30 km, and the height of the level separating the stratospheric and tropospheric ozone should be examined. From figure 8 where the weighting functions are shown, we see that as much as 20 percent of atmospheric emission could come from the region about 30 km and about 10 percent from the troposphere. By assuming photochemical equilibrium above 35 km, we may be able to reduce the emission of the top layer by half. Even then, small errors arising from these assumptions will be magnified in the solution obtained, as shown earlier.

There can be errors introduced from the incorrect temperature and dewpoint temperature soundings. Since about 50 percent of the emission from the ground is transmitted to the top, we need to know the ground temperature and emissivity quite accurately.

From the foregoing study we are inclined to believe that, at the stations where radiosonde data are available, total ozone could be estimated to about 10- to 20-percent accuracy, provided the satellite-measured radiance is accurate to 1 percent in the 9.6μ region. Since the 9.6μ band contains about 30 spectral intervals of 5 cm^{-1} width, we could use an overdetermined system of 30 equations to solve for our two unknowns. Such an overdetermined system could be adapted to a standard least-squares technique to minimize the influence on the solution of random errors in radiance measurements.

Because of the limited information content in the 9.6μ spectral region and the availability of climatological data on zone distribution, one may think of using the empirical orthogonal functions (see Holmström, 1963) to approximate the distribution. Williamson (1968) has done such a study and finds that the first two empirical orthogonal functions alone account for at most 65 percent of the variance in the sample studied. This may not offer any improvement over the two-parameter model of ozone adopted in this study. This is because of the extra information—namely the assumption of photochemical equilibrium at high altitudes and the uniform ozone distribution below tropopause—built into the model.

The indirect determination of ozone by the method developed in this study is barely sufficient to resolve the day-to-day changes, of about 10 to 20 percent, in the atmospheric total ozone over midlatitudes. It would be interesting to see how much information concerning the atmospheric ozone could be derived from the forthcoming satellite data. The indirect determination of ozone may be made more accurate by improving the spectral resolution of the radiance measured.

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